$$(p^n-1)^2+(p^n-1)p^n+p^{2n}=3p^{2n}-3p^n+1$$

is the total number of sets making $D'=0$. Subtracting this number from p^{3n}, we obtain $(p^n-1)^3$ as the order of H for $p>3$.

The group* H is an invariant subgroup of G. In view of the preceding results, the order of the quotient-group is

$$p^n(p^{2n}-1) \text{ if } p>3 \text{ or if } p=2;\ 3^{2n}(3^n-1) \text{ if } p=3.$$

This result for p not equal to 3 is in accord with the general theory of group-matrices† by which the quotient-group is seen to be simply isomorphic with the group of binary substitutions of determinant unity in the $GF[p^n]$. The latter is known to be simple if $p=2$, and to have the factors of composition $\frac{1}{2}p^n(p^{2n}-1)$ and 2 if $p>2$.

The University of Chicago, March, 1902.

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*The group H is evidently simply isomorphic with the commutative group of ternary linear transformations whose general matrix is

$$\left\{\begin{matrix} 1 & 2\alpha & 3\gamma \\ \alpha & 1+\alpha & 3\gamma \\ \gamma & 2\gamma & 1+2\alpha \end{matrix}\right\}$$

†Frobenius, Burnside, Dickson. See the references in the *Transactions of the American Mathematical Society*, July, 1902.

"THE BETWEENNESS ASSUMPTIONS."

By DR. ELIAKIM HASTINGS MOORE.

Amongst mathematicians there is abiding interest in the foundations of geometry—at present, in particular, as to the projective axioms. These axioms constitute, for instance, the first two groups I, II of Hilbert's system of axioms.

In a paper "On the Projective Axioms of Geometry," published January, 1902, in The Transactions of the American Mathematical Society (vol. 3, pp. 142-158), I exhibited and developed a new system of projective axioms for geometry of three or more dimensions, comparing it with the systems of Pasch, of Peano, and of Hilbert, and in this connection proving the redundancy in Hilbert's system I, II of the axioms I 4, II 4.

In the April 1902 number of THE MONTHLY (pp. 98-101), under the title, "The Betweenness Assumptions," Dr. Halsted published a second proof of the redundancy of II 4, a proof due to Mr. R. L. Moore, a student of his. Dr. Halsted alluded to my earlier proof of the theorem in the statement: "Mr. Moore has no intimation that any one has ever tried to prove these theorems." (l. c. p. 100.)

I wrote to Mr. Moore explaining the situation, and congratulating him upon the beauty of his proof. The congratulatory part of this letter appears in an editorial note (p. 148) of the May 1902 number of THE MONTHLY.

The letter was as follows:

THE UNIVERSITY OF CHICAGO, May 6, 1902.

MR. R. L. MOORE, The University of Texas, Austin, Texas.

MY DEAR MR. MOORE: I read with much interest, the other day, your proof of the redundancy of Hilbert's axiom II 4, in his system I, II, as exhibited by Professor Halsted in the current number of the AMERICAN MATHEMATICAL MONTHLY. Today I received from Professor Halsted a copy of that number. This is in response to a letter I sent him a week or so ago stating that I should be pleased to receive for publication in the Transactions the delightfully simple proof of the redundancy of which he wrote [had written] to me. I certainly agree with him in this estimate of your proof. Apparently he has not called your attention to the fact that the redundancy was pointed out by me and proved in my paper, which I am sending under separate cover, on the projective axioms of geometry, published in the January number of the Transactions. In accordance with correspondence with him, it was in connection with this paper of mine that he wrote to Hilbert and received Hilbert's response which led to your work on the subject. You will see that it was my desire to survey the whole system of projective axioms, and to exhibit a new system, and, in that connection to show that Hilbert's axioms I 4 and II 4 were in his system redundant, and, moreover, to furnish a satisfactory account of the rôles of the axioms I 3, 4, 5 which had been held by Schur to be redundant. As to the axiom II 4, you will see that, by considerations of the other linear axioms alone, and so in particular without the use of II 5, or of my axiom 4, I prove on page 151 that the axiom II 4 is a result of the statement 2_1, which [statement] is the statement of your theorem I. Thus to complete the proof of the redundancy of II 4, in Hilbert's system, I should today make use of your proof of theorem I. The proof that I give, in that it involves my triangle transversal axiom 4, is necessarily much longer.

I have supposed that you might be interested in understanding how your paper impresses me, and remain with considerable interest in the progress of your mathematical career,

Yours very truly,

(Signed) E. H. MOORE.

I suppose that the letter as a whole may be of value and interest to some readers of THE MONTHLY.

Chicago, June 3, 1902.

A NON-EUCLIDEAN GEM.

By DR. GEORGE BRUCE HALSTED.

La Géométrie non-euclidienne. Par P. Barbarin. Paris, C. Naud. 1902.

It is peculiarly appropriate that from Bordeaux, made sacred for non-Euclidean geometry by Hoüel, should emanate this beautiful little treatise, decor-